2025 BLOOMINGTON GEOMETRY WORKSHOP

Laura DeMarco
Title: The (algebraic) geometry of the Mandelbrot set
Abstract: One of the most famous – and still not fully understood – objects in mathematics is the Mandelbrot set. By definition, it is the set of complex numbers c for which the recursive sequence defined by $x_1 = c$ and $x_{n+1} = (x_n)^2 + c$ is bounded. This set turns out to be rich and complicated and related to many different areas of mathematics. I will present an overview of what's known and what's not known about the Mandelbrot set, and I'll describe recent work that (perhaps surprisingly) employs tools from number theory and arithmetic geometry.

Steven Frankel
Title: Quasigeodesic and pseudo-Anosov flows
Abstract: Quasigeodesic and pseudo-Anosov flows are defined by simple geometric conditions and interact in surprising ways with the geometry of 3-manifolds. I will discuss recent work with Michael Landry on this interaction, including the resolution of Calegari's Flow Conjecture, that every quasigeodesic flow on a closed hyperbolic 3 -manifold may be deformed into a flow that is both quasigeodesic and pseudo-Anosov.

Beibei Liu
Title: Rigidity of convex cocompact diagonal actions
Abstract: Convex subsets in higher-rank symmetric spaces are pretty rigid compared to rank 1 symmetric spaces, as proved by Kleiner and Leeb. In this talk, I will talk about convex subsets in products of negatively curved Hadamard manifolds. In particular, we show the limit cone is 1-dimensional if the diagonal action is convex cocompact, which induces some rigidity type of results of the diagonal representation.

Seraphina Lee
Title: Lefschetz fibrations with infinitely many sections
Abstract: A Lefschetz fibration $M^4 \to S^2$ is a generalization of a surface bundle which also allows finitely many nodal singular fibers. The Arakelov–Parshin rigidity theorem implies that nontrivial, holomorphic Lefschetz fibrations of genus $g \geq 2$ admit only finitely many holomorphic sections. In this talk, we will show that no such finiteness result holds for smooth or symplectic sections by giving examples of genus-g ($g \geq 2$) Lefschetz fibrations with infinitely many homologically distinct sections. We will also discuss examples with infinitely many orbits of sections under the action of fiberwise diffeomorphisms of M that preserves the set of fibers of $M \to S^2$. This is joint work with Carlos A. Serván.

Yuping Ruan
Title: Absolute continuity of stationary measures
Abstract: Let f and g be two volume preserving, Anosov diffeomorphisms on the 2-torus, sharing common stable and unstable cones. For any probability measure μ supported in a neighborhood of f and g, and verifying certain conditions, we show

that the unique μ-stationary SRB is absolutely continuous with respect to the ambient Haar measure. The proof is motivated by the "transversality arguments" in Tsujii's work on partially hyperbolic endomorphisms. This talk is based on a joint work with Aaron Brown, Davi Obata and Homin Lee, `https://arxiv.org/abs/2409.18252`

George Domat
Title: Classification of Stable Surfaces with respect to Automatic Continuity
Abstract: Topological groups often exhibit lots of interplay between their algebraic and topological structures. A stark example of this is the automatic continuity property: A topological group has the automatic continuity property if every (algebraic) homomorphism to any other separable group is continuous. We provide a complete classification of when the homeomorphism group of a stable surface has the automatic continuity property. Towards this classification, we provide a general framework for proving automatic continuity for groups of homeomorphisms. This is joint work with Mladen Bestvina and Kasra Rafi.

Karen Butt
Title: Monotonicity of Liouville entropy along the Ricci flow
Abstract: We consider the geodesic flow of a closed negatively curved surface. Its Liouville entropy is a complete invariant of the measurable dynamics, roughly capturing the average exponential divergence of nearby trajectories. For negatively curved surfaces of fixed total area, Katok proved this invariant is maximized at hyperbolic metrics, ie, metrics of constant negative curvature. Our main result is that, in this setting, the Liouville entropy is monotonically increasing along the normalized Ricci flow on the space of metrics. This answers affirmatively a question of Manning, and gives a new proof of Katok's aforementioned result. In addition to geometric and dynamical methods, our proof also uses microlocal analysis. This is joint work with Erchenko, Humbert, and Mitsutani.

Laura DeMarco
Title: From Manin-Mumford to Dynamical Rigidity
Abstract: In the early 1980s, Raynaud proved a theorem (the Manin-Mumford Conjecture) about the geometry of torsion points in abelian varieties, using number-theoretic methods. Around the same time, and with completely different methods, McMullen proved a dynamical rigidity theorem for holomorphic maps on P^1. In recent work, joint with Myrto Mavraki, we explained how to view these results as special cases of a unifying conjecture. (The conjectural statement is inspired by a recent theorem of Gao and Habegger, called "Relative Manin-Mumford", and results in complex dynamics of Dujardin, Gauthier, Vigny, and others.) I will present the conjecture and discuss recent progress.