2016 BLOOMINGTON GEOMETRY WORKSHOP

TITLES AND ABSTRACTS

Danny Calegari, Building Surfaces (Colloquium Talk)

Abstract: Surfaces are one of the fundamental objects in mathematics; they lie at the intersection of geometry, topology, complex analysis and group theory. Even better than surfaces are minimal surfaces — those which are the simplest, or the smallest, for the job they have to do. I will talk about what "minimal" means in topology and group theory, and how solving a topological minimization problem can give rise to geometry.

Danny Calegari, Scl, sails and surgery

Abstract: We describe an intimate connection between the theory of scl in free groups and integer programming. So-called "surgery families" of chains give rise to natural families of integer programming problems, and consequently scl is eventually a ratio of quasipolynomials on such families. This is partly joint work with Alden Walker.

Jeff Diller, Degree growth for iterates of plane rational maps

Abstract: A plane rational self-map $f : \mathbf{P}^2 \to \mathbf{P}^2$ is given in homogeneous coordinates by $f = (f_1, f_2, f_3)$, where the components f_j are homogeneous polynomials of the same degree $\deg f$, which we call the 'algebraic degree' of f. The exponential growth rate of the integer sequence $\deg f^n$ is a fundamental quantity for understanding ergodic theory of f. It can be surprisingly hard to compute, however, and understanding it in special cases uncovers some surprising connections with e.g. geometric group theory and non-archimedean geomtry. I will survey some of what is known and conclude by describing some recent results with Jan-Li Lin and some open problems concerning the degree sequence associated to a map with an invariant two form.

Wouter van Limbeek, Rigidity of convex divisible domains in flag manifolds

Abstract: A projective structure on a manifold is a local modeling of the geometry on the geometry of projective space. Projective structures usually lack rigidity: E.g. any hyperbolic manifold is canonically projective, but oftentimes the structure can be deformed. There are also projective structures on other manifolds altogether. A natural generalization of these structures is obtained by modeling the local geometry on other Grassmannians. In contrast to the plethora of examples of projective structures, we establish rigidity in this new context: We prove that in the Grassmannian of p-planes in $\mathbb{R}^{2p}$, $p > 1$, every bounded convex domain with a compact quotient is a symmetric space. This is joint work with Andrew Zimmer.

Priyam Patel, Building metrics for closed curves and applications to lifting curves simply

Abstract: It is a well known result of Peter Scott that the fundamental groups of surfaces are subgroup separable. This algebraic property of surface groups also has important topological implications. One such implication is that every closed curve on a surface lifts to a simple closed curve in a finite cover of the surface (i.e. lifts simply). A natural question that arises is: what is the minimal degree of a cover in which a given closed curve lifts simply? We will begin this talk by discussing various results answering the above question for hyperbolic surfaces. In particular, we will focus on recent joint work with T. Aougab, J. Gaster, and J. Sapir in which we build a hyperbolic metric for a closed curve on a surface that yields an interesting relationship between its length and self-intersection number. We will highlight how such a relationship can help answer the original question about lifting curves simply in finite covers.

Roland Roeder, Rational maps of $\mathbb{CP}^2$ with equal dynamical degrees, no invariant foliation, and two distinct measures of maximal entropy

Abstract: The ergodic properties of a rational map $f : \mathbb{CP}^2 \to \mathbb{CP}^2$ are tied to its dynamical degrees $\lambda_1(f)$ and $\lambda_2(f)$. Maps with $\lambda_1(f) > \lambda_2(f)$ share many properties of the Hénon maps, having a measure of maximal entropy of saddle type. Maps with $\lambda_2(f) > \lambda_1(f)$ share many properties of holomorphic endomorphisms, having a measure of maximal entropy that is repelling. In both cases it is believed (and often proved) that f has a unique measure of maximal entropy.

Early examples of maps with $\lambda_1(f) = \lambda_2(f)$ were skew products, having an invariant fibration. Guedj asked whether this happens in general. We show that there is a simple way to produce many rational maps of $\mathbb{CP}^2$ with equal dynamical degrees, no invariant foliation, and two measures of maximal entropy, one of saddle-type and one repelling. Many of the techniques are geometric. This is joint work with Jeff Diller and Han Liu, and it builds on previous joint work with Scott Kaschner and Rodrigo Pérez.

Giulio Tiozzo, Counting hyperbolic elements via random walks

Abstract: Let us consider a group G of isometries of a delta-hyperbolic metric space X, which is not necessarily proper (e.g. it could be a locally infinite graph). We can define a random walk on G by picking random products of elements of G, and projecting this sample path to X.

We show that such a random walk converges almost surely to the Gromov boundary of X, and with positive speed. We then apply these techniques to counting hyperbolic elements with respect to balls in the Cayley graph.

This is joint work with J. Maher, I. Gekhtman and S.Taylor.

Susan Tolman, Non-Hamiltonian symplectic circle actions with isolated fixed points.

Abstract: A circle action on a symplectic manifold is "symplectic" if it preserves the symplectic form and "Hamiltonian" if there exists a moment map. In the latter case, many invariants of the manifold are determined by the fixed set. Therefore, it is important to determine when symplectic actions are Hamiltonian. We answer a question posed by McDuff and Salamon by constructing a non-Hamiltonian symplectic circle action with exactly 32 fixed points on a closed-connected, six-dimensional symplectic manifold.

Shi Wang, Barycentric straightening and bounded cohomology

Abstract: In this talk, I will report on joint work with Jean Lafont. We show that, for an n-dimensional irreducible symmetric space of rank $r \geq 2$ (excluding $SL(3, \mathbb{R})/SO(3)$ and $SL(4, \mathbb{R})/SO(4)$), the p-Jacobian of barycentrically straightened simplices has uniformly bounded norm, provided $p \geq n - r + 2$. As a consequence, for the corresponding non-compact, connected, semisimple real Lie group G, every degree p cohomology class has a bounded representative. This answers Dupont's problem in small codimension. We also give examples of symmetric spaces where the barycentrically straightened simplices of dimension $n - r$ have unbounded volume, showing that the range in which we obtain boundedness of the p-Jacobian is very close to optimal. I will also discuss some of my recent work on its generalization.