

# Bloomington Geometry Workshop 2015

## Titles and Abstracts.

1. Benson Farb, (Colloquium Talk) Spaces of polynomials: from 1799 to today (or, "Everything I needed to know I learned in Kindergarten and from EGA")

Why can't we solve the quintic? Why is it so hard to find, or even get close to finding, roots of polynomials? Why does the number of linear factors of a degree  $d$ , square-free polynomial over a finite field stabilize for large  $d$ ? In this talk I will explain how these and many other such questions are a reflection of the topology of various spaces of polynomials. The level of mathematics here ranges from among the most basic (the quadratic formula) to among the most sophisticated (etale cohomology and the theory of motives), each point-of-view informing the other. Parts of this talk represent joint work with Thomas Church and Jordan Ellenberg, and also with Jesse Wolfson.

2. Benson Farb, Exotic hyperbolic geometry, homological vanishing, and the  $p$ -Jacobian of the barycenter map.

In this talk I will explain a homology vanishing theorem for infinite volume, exotic-hyperbolic manifolds. The key ingredient is a new estimate on the  $p$ -Jacobian of the barycenter map, a remarkable map defined by Besson-Courtois-Gallot. This is joint work in progress with Chris Connell and Ben McReynolds.

3. Michael Khanevsky, Hofer length spectrum of symplectic surfaces

We will discuss geometry of Hamiltonian diffeomorphisms in symplectic surfaces with respect to the Hofer metric. Despite of very elementary setup much in this subject is still unexplored. In Riemannian geometry the length spectrum is defined as the minimal length of a closed curve in each homotopy class. It is a rich source of invariants of the manifold. In symplectic setting there is no notion of length, hence no possibility to define the usual length spectrum. However, one can construct a similar invariant by measuring Hofer's length of closed 'trajectories' of disks. We will provide some estimates for this construction.

4. Sarah Koch, Galois Invariant Sets in Dynamics.

In William Thurston's last paper, *Entropy in Dimension One*, he completely characterizes which numbers arise as  $\exp(\text{entropy}(f))$ , where  $f$  is a postcritically finite real map of a closed interval. On page 1 of this paper, there is a spectacular image of a Galois invariant set associated to entropy values of quadratic polynomials. This set displays some amazing fractal structure which can be (somewhat) understood when viewed as a subset of parameter space for a particular family of iterated function systems (IFS). We compare this with the parameter space discussion of the family  $z \mapsto z^2 + c$ , and investigate the associated connectedness locus in parameter space for the IFS. If time permits, we study another Galois invariant subset arising in the dynamical realm which displays similar structure; this set is also associated to postcritically finite quadratic polynomials. Parts of this talk are joint work with D. Calegari and A. Walker, and parts of this talk are joint work with X. Buff and A. Epstein.

5. Jean-François Lafont, Combinatorial vs. Riemannian structures on manifolds.

A combinatorial structure on a smooth manifold is a smooth triangulation of the manifold. I will describe combinatorial analogues of the classical systolic inequalities in Riemannian geometry. Some applications will be given. This is joint work with Ryan Kowalick and Barry Minemyer.

6. Andy Putman, Integrality in the Steinberg module and the top-dimensional cohomology of  $\text{SL}_n(O_K)$ .

I will discuss the relationship between the arithmetic of a number ring  $O_K$  and high-dimensional cohomology of  $\text{SL}_n(O_K)$ . This is joint work with Tom Church and Benson Farb.

7. Ralf Spatzier, Higher Rank Rigidity and Positive Curvature.

I will review rigidity and non-rigidity results about "higher rank" in Riemannian geometry. Specifically we consider higher rank spaces in which subobjects of extremal curvature are plentiful. I will emphasize recent joint work with Schmidt and Shankar on Riemannian manifolds of higher spherical rank where every geodesic  $c$  has a perpendicular parallel field making sectional curvature 1 with  $c$ , and the sectional curvature is bounded below by 1.

8. Yi Wang, Integral conformal invariants.

In this talk, I will discuss integral conformal invariants on the boundary of four dimensional manifolds. This includes boundary curvature terms in the Chern-Gauss-Bonnet formula for manifolds with boundary. I will also talk about geometric properties for manifolds with totally finite conformal invariants.

9. Andrew Zimmer, Rigidity and flexibility of symmetric domains.

Many symmetric spaces of non-compact type can be embedded as a domain in some compact homogeneous space. For instance, real hyperbolic space embeds as a domain in real projective space and the projective symmetries of the domain coincide with the isometry group (this is the Klein model). This embedding is quite flexible in that there exists perturbations which are non-symmetric but still have large projective symmetry group (for instance co-compact). Perhaps more surprisingly, in real projective space there also exist non-symmetric domains whose projective symmetry group is discrete, co-compact, and not quasi-isometric to any symmetric space. In this talk we will survey some recent rigidity results for domains in other compact homogeneous spaces (complex/quaternionic projective space and certain Grassmannians). One of the results we will discuss is joint work with Wouter van Limbeek.