

# Bloomington Geometry Workshop 2012

## Titles and abstracts.

- (1) Mladen Bestvina, Topology of Numbers.

The first half of the talk will be a review of how to solve, following Conway, equations such as  $x^2 - 2 * y^2 = 6$  over the integers. Roughly speaking, one enters a number (value of the form) in a few complementary components of the  $SL(2, \mathbb{Z})$  spine (=topograph) in hyperbolic plane, and the solutions appear. Positive and negative values are separated by a line (=river), which is the axis of the symmetry group. The second half of the talk will sketch a generalization of this to hyperbolic 3-space, corresponding to integral binary hermitian forms. The topograph becomes the spine of the suitable Bianchi group, and the river becomes an ocean (a copy of hyperbolic plane). This is joint work with Gordan Savin.

- (2) Mladen Bestvina, Teichmuller Theory in Outer Space.

I will give an introduction to/survey of recent developments in the geometry of Outer space, a contractible complex with an action of  $Out(F_n)$ . The results are motivated by Teichmuller theory.

- (3) Will Cavendish, Towers of Covering Spaces of 3-manifolds and Mapping Solenoids.

Agol's recent resolution of the virtually Haken conjecture together with work of Wise and the tameness theorem show that any  $\pi_1$ -injective map  $f : S \rightarrow M$  of a surface  $S$  into a geometric 3-manifold  $M$  can be lifted to a map  $\tilde{f}$  into a finite sheeted covering space  $\tilde{M} \rightarrow M$  that is homotopic to an embedding. Examples of Rubinstein and Wang show, however, that there exist maps of surfaces into non-hyperbolic 3-manifolds that are not "virtually embedded" in this sense. In this talk we will discuss a construction called the mapping solenoid of  $f$ , and show how the cohomology groups of this object can be viewed as obstructions to solving topological lifting problems. We will then discuss the cohomology of the mapping solenoid associated to the Rubinstein-Wang examples and show that these obstructions do not vanish in this setting.

- (4) Jon Chaika, Quantitative shrinking targets for interval exchange transformations and rotations.

In this talk we present some quantitative shrinking target results. Consider  $T : [0, 1] \rightarrow [0, 1]$ . One can ask how quickly under  $T$  a typical point  $x$  approaches a typical point  $y$ . In particular given  $\{a_i\}_{i=1}^{\infty}$  is  $T^i x \in B(y, a_i)$  infinitely often? A finer question of whether  $T^i x \in B(y, a_i)$  as often as one would expect will be discussed. That is, does

$$\lim_{N \rightarrow \infty} \frac{\sum_{n=1}^N \chi_{B(y, a_n)}(T^n x)}{\sum_{n=1}^N 2a_n} = 1$$

for almost every  $x$ .

We will present applications to billiards in rational polygons and a related result for Sturmian sequences. This is joint work with David Constantine.

- (5) Tullia Dymarz, Lattice envelopes for groups acting on horocyclic products.

If a countable group  $G$  is a lattice in a locally compact group  $H$  then we say that  $H$  is a lattice envelope for  $G$ . In this talk we describe all possible lattice envelopes for certain solvable groups that act on horocyclic products of  $\text{CAT}(-1)$  spaces. The techniques involved come from quasi-isometric rigidity and analysis on boundaries of negatively curved spaces.

- (6) Alex Furman, Classifying lattice envelopes of some countable groups.

In a joint work with Uri Bader and Roman Sauer we study the following question: given a countable group  $\Gamma$  describe all locally compact groups  $G$  which contain a copy of  $\Gamma$  as a lattice (uniform or non-uniform). I will discuss the solution of this problem for a large class of groups. The proof relies on several deep and unexpected tools, including Breuillard-Gelander's topological Tits alternative, Margulis' commensurator superrigidity, arithmeticity, and normal subgroup theorems, quasi-isometric rigidity results of Kleiner-Leeb, Mosher-Sageev-Whyte.

- (7) Ilya Kapovich, On spectrally rigid subsets of free groups.

It is well known that any tree  $T$  in the (unprojectivized) Culler-Vogtmann Outer space  $cv_N$  is uniquely determined by the translation length function (also known as the marked length spectrum) of  $T$ ,  $\|\cdot\|_T : F_N \rightarrow [0, \infty)$ . Here for  $g \in F_N$ ,  $\|g\|_T = \inf_{x \in T} d_T(x, gx)$ . We say that a subset  $S \subseteq F_N$  is spectrally rigid in  $F_N$  if whenever  $T, T' \in cv_N$  are such that  $\|g\|_T = \|g\|_{T'}$  for every  $g \in S$  then  $T = T'$  in  $cv_N$ . By contrast to similar questions for the Teichmüller space, it is known that for  $N \geq 2$  there does not exist a finite spectrally rigid subset of  $F_N$ . We will discuss known results and open problems about spectral rigidity and non-spectral rigidity of various "natural" infinite subsets of  $F_N$ .

- (8) Thomas Koberda, Mapping class groups, homology and finite covers of surfaces.

Let  $S$  be a orientable surface of negative Euler characteristic. We will discuss the action of pseudo-Anosov mapping classes of  $S$  on the homology of various finite covers of  $S$  to which they lift. We will be particularly interested in finding a lift of each pseudo-Anosov mapping class for which the homological spectral radius is greater than one. For a given mapping class, we will relate the study of this problem to the topology of the mapping torus.