Bloomington Geometry Workshop 2007 Titles and abstracts.

1 Stephanie Alexander, Topology of nonnegatively curved Euclidean hypersurfaces spanning a prescribed boundary

We prove that a smooth compact immersed submanifold of codimension 2 in Euclidean n-space, $n > 2$, bounds at most finitely many topologically distinct compact nonnegatively curved hypersurfaces. On the other hand, we construct an example to show that this result may not hold if the boundary is not everywhere smooth. Indeed we construct a simple closed rectifiable curve in 3-space that is smooth in the complement of two points, and is differentiable at these two points with respect to the arclength parameter, but bounds infinitely many topologically distinct compact smooth positively curved surfaces. Our finiteness theorem extends to noncompact immersed spanning hypersurfaces when $n > 3$, and to noncompact embedded spanning surfaces when $n = 3$. This is joint work with Mohammad Ghomi and Jeremy Wong.

2 Robert Gulliver, Total curvature of curves and of graphs

Whenever singularities are present, there is a bifurcation of concepts. An example would be the curvature of a piecewise-smooth curve in space. The exterior angle at a corner serves for many contexts as the correct contribution of the vertex to total curvature. However, for the variation of length when the variation vector fields has length at most one, the total curvature is twice the tangent of half that angle. This bifurcation becomes more serious when the vertex has valence three or greater: one notion of total curvature is useful for bounding the density of a 2-dimensional minimal current with the given graph as boundary, and another notion is relevant to the isotopy class of the graph. This is based in part on joint work with Sumio Yamada.

3 Tadeusz Januszkiewicz, Simplicial non-positive curvature and filling invariants.

Simplicial nonpositive curvature. (based mainly on joint work with Jacek Swiatkowski)

SNPC is a combinatorial condition on links in a simplicial complex which implies many of the consequences of metric nonpositive curvature.

Many SNPC spaces exists but they – especially high dimensional ones – have properties very different from classical examples. Probably the most striking difference is the behaviour of filling invariants.

I will describe basic concepts, constructions and, time permitting, some applications of wider interest.

4 Ilya Kapovich, Geodesic currents on free groups.

A geodesic current is a measure-theoretic analogue of the free homotopy class of closed curve. Studying geodesic currents on hyperbolic surfaces proved useful in low-dimensional topology, as demonstrated by the work of Bonahon. We will discuss the differences and the similarities between the hyperbolic surface case and the case of geodesic currents on a finitely generated free group. Applications include constructing a new compactification of the Culler-Vogtmann outer space.

5 William Minicozzi, Width and finite extinction time of Ricci flow.

I will discuss a geometric invariant, the width, of a manifold and show how it can be realized as the sum of areas of minimal 2-spheres. For instance, when M is a homotopy 3-sphere, the width is loosely speaking the area of the smallest 2-sphere needed to "pull over" M. Second, we discusses an application of this involving Hamilton's Ricci flow. This is joint work with Toby Colding of MIT/NYU.

6 Liviu I. Nicolaescu, Tame Flows.

The tame flows are "reasonable" flows on "reasonable' spaces. The reasonable, or *tame*, spaces are the pfaffian varieties introduced by A. Khovanski, and a flow $\Phi : \mathbb{R} \times X \to X$ on pfaffian variety X is called *tame* if the graph of Φ is a pfaffian subvariety of $\mathbb{R} \times X \times X$. Tame geometry is a vast generalization of the better known subject of real algebraic geometry, and the first half of my talk will be a brief introduction to modern tame geometry. In the second half, I will show that there are plenty of Morse-like tame flows, and then describe how to use the tameness to draw rather nontrivial geometric conclusions.

7 Christina Sormani, Continuity of the Covering Spectra of Length Space.

The covering spectrum of a length space (or Riemannian manifold) roughly measures the size of its one dimensional holes. On compact length spaces, it has been shown to be closely related to the length spectrum and useful for determining when the space has a universal cover. It is also continuous under Gromov-Hausdorff convergence of compact spaces.

On complete spaces, I will demonstrate that the covering spectrum is not so well behaved. I will then introduce the R cut-off covering spectrum which only detects a localized collection of the holes in the space. It is closely related to the length spectrum and is continuous with respect to pointed Gromov-Hausdorff convergence when the spaces are locally compact. I then build the cut-off covering spectrum which is not localized and examine its properties. The talk will close with applications to spaces with curvature bounds. The concepts and results are new even for Riemannian manifolds. This is joint work with Guofang Wei.

8 Giuseppe Tinaglia, The rigidity of complete embedded CMC surfaces of finite genus.

We study the rigidity of complete embedded constant mean curvature surfaces in space. One of the results that we prove is the following: Let M be a complete embedded constant mean curvature surface of finite genus. If M has bounded Gaussian curvature and M is not the helicoid then M is rigid; being rigid means that the inclusion map of M into space represents the unique isometric immersion of M into space with the same constant mean curvature up to ambient isometries. A key ingredient in the proof is our recent Dynamics Theorem for CMC surfaces which will be discussed during the talk. This is joint work with Bill Meeks.

9 Frederico Xavier, Using Gauss maps to detect intersections

It is shown that a family of compact submanifolds with boundary has a non-empty intersection in $\mathbb{R}^n$ provided a certain geometric estimate holds. The inequality in question involves three features: the intrininsic sizes of the submanifolds, a weighed maesure of the effect of translations, and the distortion of the configurations of normal spaces. An application is the following global result. Let $M_1, \cdots, M_n$ be properly embedded connected smooth hypersurfaces without boundary. Consider the (unoriented) Gauss map given by $\mathcal{G}_j : M_j \to G(1, n) \cong \mathbb{RP}^{n-1}$, $\mathcal{G}_j(p) = [T_p M_j]^\perp$. If every hyperplane in $\mathbb{RP}^{n-1}$ intersects at most $n-1$ of the sets $\overline{\mathcal{G}_1(M_1)}, \cdots \overline{\mathcal{G}_n(M_n)}$, then $M_1 \cap \cdots M_n$ consists of a single point. If the time allows we will also discuss some recent applications of related ideas to several complex variables and algebraic geometry.